Problem Solving: Exponential Growth and Decay

Compound Interest Formula	Doubling-Time Growth Formula	Half-Life Decay Formula
$A = P\left(1 \pm \frac{r}{n}\right)^{nt}$	$N = N_0 2^{\frac{t}{d}}$	$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$
A: Amount at time t	N_0: Starting Amount	N_0: " "
P: Principle (Amount you start w/).	N: Amount at time t.	N: " "
r: Annual interest rate (in decimal form	t: time (any units)	t: " "
	d: doubling time (same units as t).	h: half life (amount of time it takes for the substance to decay by half)
n: number of times interest rate is compounded annually	*Note: t and d must be the same units	

Example 1: $4,000 is invested at 5% interest compounded annually. (a) Determine how much the investment is worth after 3 years, (b) How much is the investment worth after 3 years when interest is compounded quarterly?

$P = 4{,}000 \quad r = .05 \quad t = 3 \text{ years} \quad n = 1$

$A = 4000\left(1 + \frac{.05}{1}\right)^{3\cdot 1} \rightarrow 4000(1.05)^3 \quad 4000(1.157625)$ ⓐ $\boxed{\$4{,}630.50}$

$4000\left(1 + \frac{.05}{4}\right)^{3(4)}$

$4000(1 + .0125)^{12}$

$(1.0125)^{12}$

$4000(1.1607)$

$\boxed{\$4{,}643.02}$

Example 2: I bought my grandma's car from her for $3,000 dollars. The value decreases at a rate of about 20% per year. I have had the car for 4 years. How much is it worth now?

$A = 3000\left(1 - \frac{.2}{1}\right)^4 = \boxed{\$1{,}228.8}$

Example 3: A population of zombies doubles every 2 weeks (provided there are enough humans for them to feed on). The number of zombies in the population now is 77. Find the size of the zombie population in 18 weeks (about the time when school ends). $N = 77(2)^{18/2} = \boxed{39{,}424}$

Example 4: Superman needs some help with a math problem that he is terribly interested in for some reason, here it is: Kryptonite has a half-life of 250,000 years. If the amount of kryptonite in the entire universe is N_0, find the amount of kryptonite that will remain in 2,000 years.

$N_0\left(\frac{1}{2}\right)^{\frac{2000}{250{,}000}}$

$\left(\frac{1}{2}\right)^{.008}$

$\boxed{N_0(.99)}$