

The Complex Numbers

Recall: $\sqrt{-1} = i$ and $i^2 = -1$ by definition.

Simplify

Example

① $i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$ ② $i^{12} = (i^2)^6 = (-1)^6 = 1$

③ $i^{75} = i^{74} i^1 = (i^2)^{37} i = (-1)^{37} (i) = 1(i) = i$

Note: When you have i to any even exp. you can write it as $(i^2)^{\dots}$ ← another exponent.

When you have an odd exponent, you can "take out" a single i to make the rest even and deal with the even.

Complex Numbers are numbers that include both Real and Imaginary numbers. They have the form $a + bi$ where a and b are both real numbers.

Note: Sums and products of complex numbers work like sums and products of binomials.

Simplify

Example

Simply treat i as if it were a variable! ... until you get i^2 , then replace it with -1

④ $(3 + 6i) - (4 - 2i)$
 $3 + 6i - 4 + 2i$
 $-1 + 8i$

⑤ $(3 + 4i)^2$
 $(3 + 4i)(3 + 4i)$

3	9	12i
4i	12i	16i ²

 $9 + 12i + 12i + 16i^2$
 $9 + 24i + 16(-1)$
 $9 - 16 + 24i$
 $-7 + 24i$

⑥ $(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7})$
 $(\sqrt{3} + i\sqrt{7})(\sqrt{3} - i\sqrt{7})$

$\sqrt{3}$	3	$i\sqrt{21}$
$-i\sqrt{7}$	$-i\sqrt{21}$	$-7i^2$

 $3 - 7i^2$
 $3 - 7(-1)$
 $3 + 7$
 10

Rationalize the denominator by multiplying the top and bottom by the complex conjugate

⑦ $\frac{2+i}{6+2i} \cdot \frac{6-2i}{6-2i}$

6	12	$6i$
$-2i$	$-4i$	$-2i^2$

 $12 + 6i - 4i - 2i^2$
 $12 + 2i + 2$
 $14 + 2i$

6	36	$12i$
$-2i$	$-12i$	$-4i^2$

 $36 + 4$
 40

$\frac{14+2i}{40}$
 $\frac{7+i}{20}$
 $\frac{7}{20} + \frac{i}{20}$

⑧ $(2 - i\sqrt{3})(2 + i\sqrt{3})$

2	4	$-2i\sqrt{3}$
$i\sqrt{3}$	$2i\sqrt{3}$	$-i^2 \cdot 3$

 $4 - (-1)(3)$
 $4 + 3 = 7$